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Jet diffusion from a circular nozzle above a solid plane

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The decay of a jet discharging from a circular nozzle parallel to and displaced from a solid surface is investigated under conditions where the transitional process from circular-jet flow to oblate wall-jet flow begins in the initial, transition or self-preserving regions of the original jet. The influence of displacement of the nozzle from the plane on the developed three-dimensional wall jet downstream is demonstrated and it is found that the transitional interaction with the plane is more extended when the plane interacts first in the initial zone of the circular jet. Measurements of turbulence and Reynolds stress show the transverse mixing parallel to the plane to exceed that perpendicular to the plane, and are generally consistent with the spreading rates in these two directions, the ratio of which approaches 8·5 at large distances from the nozzle. It is shown that the interaction between the plane and jet involves a relatively large-scale coherent motion in which components of velocity directed towards or away from the surface are associated with outflow or inflow along the surface. This motion is more extended in the direction parallel to the surface and provides a mechanism for the increases in mixing rate in the direction parallel to the plane.

1. Introduction

The investigation to be described was undertaken to provide a frame-work for the estimation of the decay and diffusion of initially circular jet exhausts discharging parallel to and displaced above an effectively infinite solid plane. Newman *et al.* (1971) investigated the mean velocity distribution far downstream of a jet discharging from a circular nozzle adjacent to a plane (i.e. $h/D = 0{\cdot}5$, where h is the distance from the nozzle centre-line to the ground plane and D is the nozzle diameter), which showed a much more rapid diffusion parallel to the plane than at right angles to the plane. It is the purpose of the present work to investigate the influence of the distance of the nozzle from the plane (i.e. $h/D > 0{\cdot}5$). Also it is our intention to investigate the structure of the turbulent flow in more detail with a view to explaining the cause of the substantial difference in diffusion rates transverse to the mean flow and parallel to and perpendicular to the plane. The displacement (h/D) from the plane was not considered by Newman *et al.* (1971). However, it is of considerable relevance in the prediction of the behaviour of aircraft jet exhausts and their loading effects on ground structures. Further measurements of diffusion from a nozzle over a plane were made by Rajaratnam & Pani (1972), but again measurements were only made with the nozzle immediately adjacent to the plane.

2. Mean velocity distribution

Experiments were carried out using a nozzle of diameter 2·54 cm supplied via a smooth contraction from a settling chamber with a contraction ratio of 16 : 1. Honeycomb and gauze screens were inserted into the chamber to reduce the turbulence level in the core. The nozzle had a sharp edge, and schlieren photographs (Davis 1971) showed the jet to have a very short laminar column followed by a short distance ($\sim 0{\cdot}1D$) in which regular disturbances were observed before the shear layer became fully turbulent. It thus appeared that the shear layer was not influenced by any small residual turbulence in the potential core. The jet was operated at a speed of 100 m s^{-1}. Velocity measurements were made using linearized hot-wire anemometers together with 2 mm long, 5 μm diameter tungsten wires. A single normal wire was used for the majority of axial component velocity measurements, whilst cross-wire probes with two wires at 45° to the mean flow were used for transverse-turbulence and Reynolds-stress measurements by means of sum and difference amplifiers. The ground plane was surfaced with a smooth plastic laminate and was made to be flat to within 0·002 m over its total length of 1·7 m.

The hot-wire anemometers were of constant resistance type having a frequency response of approximately 20 KHz as described by Davis (1970). They were fitted with 10 point diode chain linearizers and were calibrated and found to deviate from linear response by less than 1 % over the range of velocities used. The anemometer units have been described in detail by Davies, Davis & Wold (1966). The sensitivity of the hot-wire to inclined incident flow has been discussed in detail by Davis & Davies (1972). The outputs from the two wires were fed to sum and difference amplifiers to form u and v component signals and were calibrated directly in the jet for u and v sensitivity. Particular laws for inclined-wire sensitivity were not therefore introduced. The calibrations were checked by comparison of streamwise turbulence levels with those measured elsewhere (e.g. by Wygnanski & Fiedler 1969) and using a wire mounted perpendicular to the flow. Also, the transverse sensitivity was checked by comparison of the indicated maximum Reynolds stress in the free jet with that of Wygnanski & Fiedler (1969). As discussed by Morrison, Perry & Samuel (1972), the Reynolds stress measurement is a convenient way of carrying out a check on inclined-wire calibrations. These calibration checks indicated the cross-wire to be accurate to better than 5 % at maximum speed. This precision is likely to deteriorate to around 10 % of the local value for measurements made at the outer edge of the jet where the local turbulence intensity is high.

The general structure of the flow is illustrated in figure 1. For a certain distance from the nozzle exit, depending upon the clearance h/D, the jet will behave in a manner basically similar to a free unbounded jet having an initial zone where a smooth core flow is surrounded by a progressively thickening turbulent shear layer as described by Davies, Fisher & Barratt (1963) or Bradshaw, Ferris & Johnson (1964) for example. However, at a distance depending upon the clearance, the flow and ground plane will begin to interact as the jet diffuses to meet the plane, and a new structure of flow will develop. For large clearances (h/D) this commencement of interaction will take place

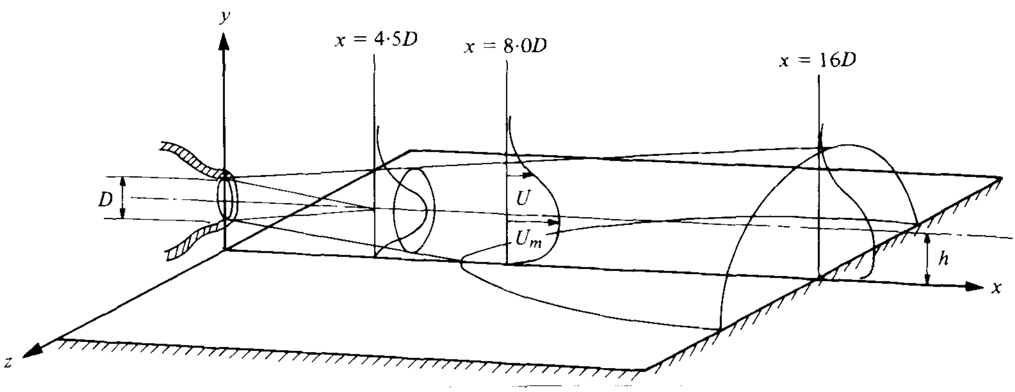
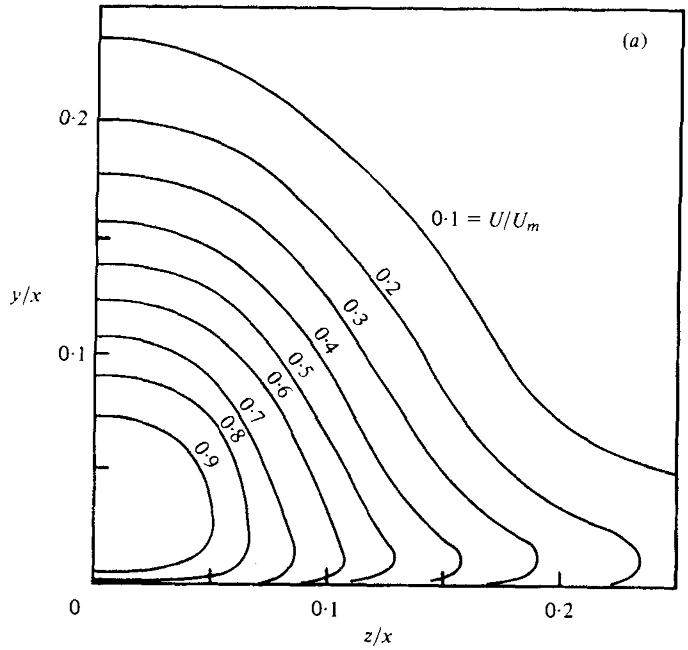


FIGURE 1. General geometry of interacting circular jet and plane. Velocity profiles shown for $h/D = 1 \cdot 0$.

between a fully turbulent jet having the characteristics described by Wygnanski & Fiedler (1969). For smaller clearances the first jet/plane interaction will be either in the transitional region between the fully turbulent and initial free-jet zones or, for very small clearances, in the initial zone itself. In approximate terms if we regard the transition region of a free jet as extending over $5 < x/D < 16$, and define the outer 'edge' of a free jet as the point where $U/U_m = 0 \cdot 1$ (U being the local mean velocity and U_m the maximum at a given distance x from the nozzle exit), then the plane initial interaction will be with a fully turbulent self-preserving jet for $h/D > 2 \cdot 6$ approximately. For $1 \cdot 2 < h/D < 2 \cdot 6$ the interaction will commence in the transition region of the free jet whilst for $h/D < 1 \cdot 2$ it will commence in the initial region of the free jet. Of course these values depend upon what is taken as the edge of a free jet, but suggest that if the jet/plane interaction is investigated over the range $0 \cdot 5 < h/D < 4$ as in the present work then each of the different types of initial interaction will be encountered. For this reason values of $h/D = 0 \cdot 5$, $1 \cdot 0$, $2 \cdot 0$ and $4 \cdot 0$ were adopted.

Different stages in the interaction process are shown by the mean velocity distributions over the cross-section of the flow in figure 2, where results of the present work are compared with those of Newman *et al.* (1971). The development of the more rapid sideways diffusion across the plane appears to begin most rapidly in the outer low-velocity regions of the mixing flow. This gives rise to regions where the equal velocity contours are curved away from the plane of symmetry. At very large distances downstream the details of the nozzle geometry including its spacing from the plane would not be expected to influence the observed profiles. Thus the results of figure 2 illustrate transitional stages from the original free jet to a self preserving three-dimensional wall jet. The transitional process in this case appears to be very much longer than for a free jet. This might be expected as the flow is required to change its cross-sectional shape from circular to a very elongated form. We see that in the fully developed three-dimensional wall jet at large distances from the nozzle the jet has a transverse dimension parallel to the plane which greatly exceeds the dimension perpendicular to the plane. Figure 1 illustrates the transition from a free jet initially uninfluenced by the plane to the three-dimensional wall jet, the velocity maximum moving towards the plane and a steep velocity gradient forming close to the surface of the plane.



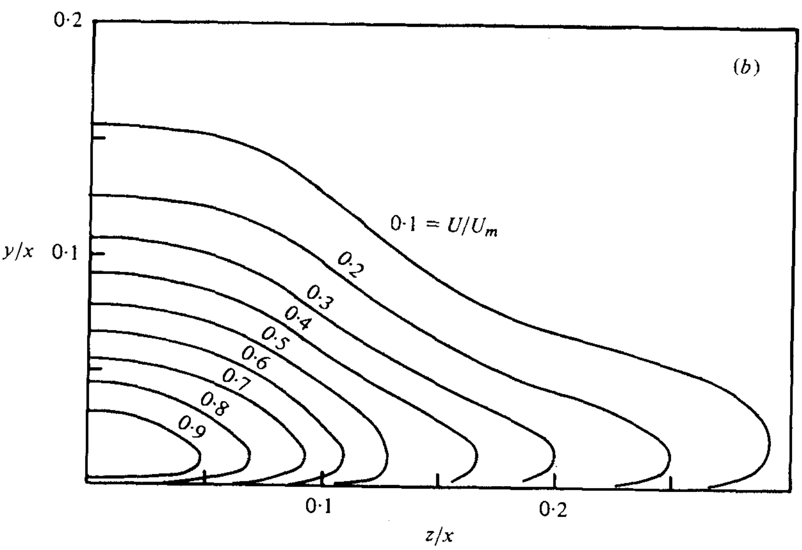
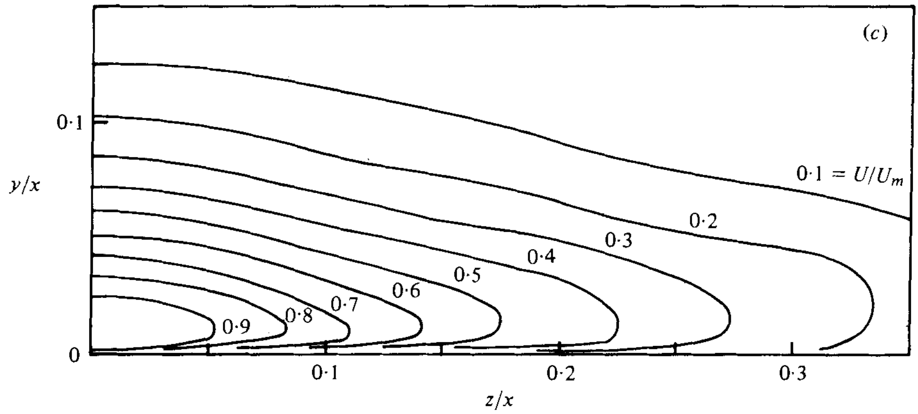


FIGURE 2 (*a*, *b*). For legend see facing page.

The decay of the maximum velocity with x/D is modified by the presence of the ground plane as shown in figure 3, the local maximum velocities being increased compared with those an equivalent distance from the free-jet case. It may be seen that as the clearance h/D is increased the zone of initial interaction moves away from the nozzle exit and the apparent origin of the jet in terms of the maximum velocity also moves away from the exit plane. The axial distributions of maximum mean velocity may be collapsed in terms of distance x_i and velocity U_{mi}, where suffix i denotes the mid-point of the interaction transition from undistorted circular to



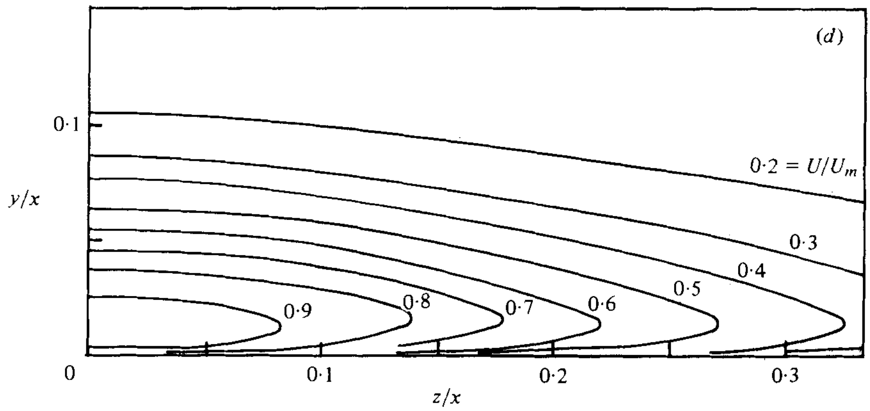


FIGURE 2. Contours of equal velocity on cross-section of flow $x/D = 48$, $Re_j = 1{\cdot}7 \times 10^5$. (*a*) $h/D = 4{\cdot}0$. (*b*) $h/D = 2{\cdot}0$. (*c*) $h/D = 0{\cdot}5$. (*d*) Results of Newman *et al.*'s (1971) profile at $x/D = 150$, $h/D = 0{\cdot}5$.

three-dimensional wall jet. This point is chosen as the position where the value of U_j/U_m is mid-way between the undisturbed circular jet value at this position and the asymptotic relation for U_j/U_m,

$$U_j/U_m = 0{\cdot}15(x/D - x_0/D).$$

Table 1 shows the values of x_0/D taken from figure 3, there being a progressive increase in the displacement of the apparent origin in maximum velocity terms as h/D is increased. The apparent origin on this basic tends towards the relatively small displacement observed for free jets (between 3 and 7 diameters, Wygnanski & Fiedler 1969) at small values of h/D. Figure 4 shows the resulting collapse of the mean velocity distributions through the interaction transitional region. The apparent origin is closely approximated by $x/x_i = 0{\cdot}35$ (dashed line). Where the first interaction takes place in the fully turbulent region of the circular jet the transition to wall-jet behaviour seems to be somewhat more rapid (solid line, for $h/D = 4{\cdot}0, 2{\cdot}0$). It should be noted that the values of x_i/D in table 1 for $h/D = 4{\cdot}0$ and $3{\cdot}0$ are estimated on the basis of an extrapolation of x_i/h, which approaches a constant value (of $17{\cdot}0$ approximately) for the larger values of h/D.

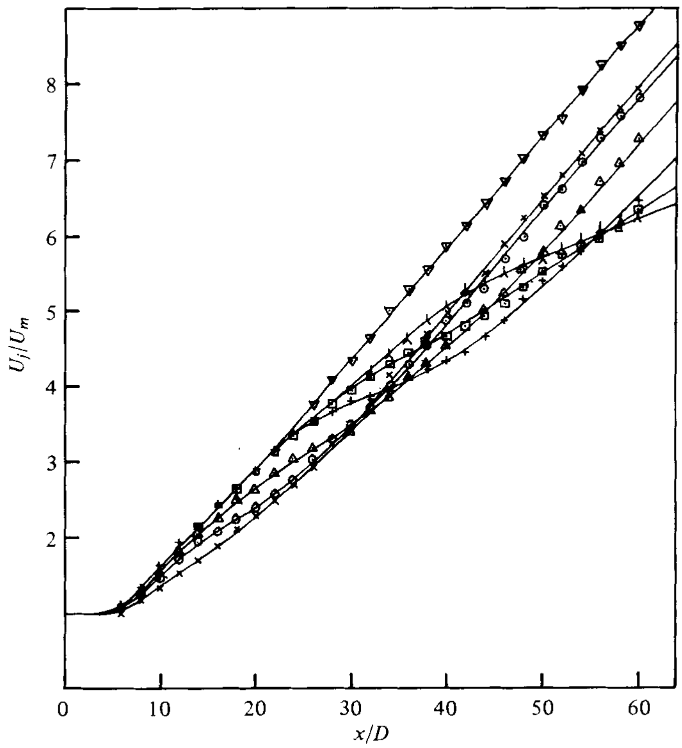


FIGURE 3. Decay of maximum velocity. ∇, $h/D = \infty$; $\curlywedge$, $h/D = 4{\cdot}0$; $\boxdot$, $h/D = 3{\cdot}0$; $+$, $h/D = 2{\cdot}0$; $\triangle$, $h/D = 1{\cdot}5$; $\odot$, $h/D = 1{\cdot}0$; $\times$, $h/D = 0{\cdot}5$.

h/D	x_0/D	x_t/D	U_j/U_{mi}
0·5	5·6	15·6	1·85
1·0	6·8	20·0	2·40
1·5	10·0	27·8	3·30
2·0	15·2	35·4	4·03
3·0	17·9	51·0	5·77
4·0	23·0	68·0	6·63

TABLE 1. Parameters of axial distribution of maximum velocity.

An interesting feature of the decay of the maximum velocity with distance from the nozzle is that the rate of decay is very close to that of a free jet. Whilst the shear on the ground plane is known to play a very small part in the overall momentum balance (Rajaratnam 1976), it appears that this aspect of the three-dimensional wall-jet decay results from the rates of increase of jet height and width which occur. Variations of these scales will be discussed in subsequent paragraphs, but it does not seem that a very simple explanation for the similar rate of decay of free and wall jets can be found.

The form of the mean velocity profile measured perpendicular to the surface and parallel to the surface does not deviate strongly from that of the free jet. This is shown in figure 5 where the profiles perpendicular to the surface have been normalized

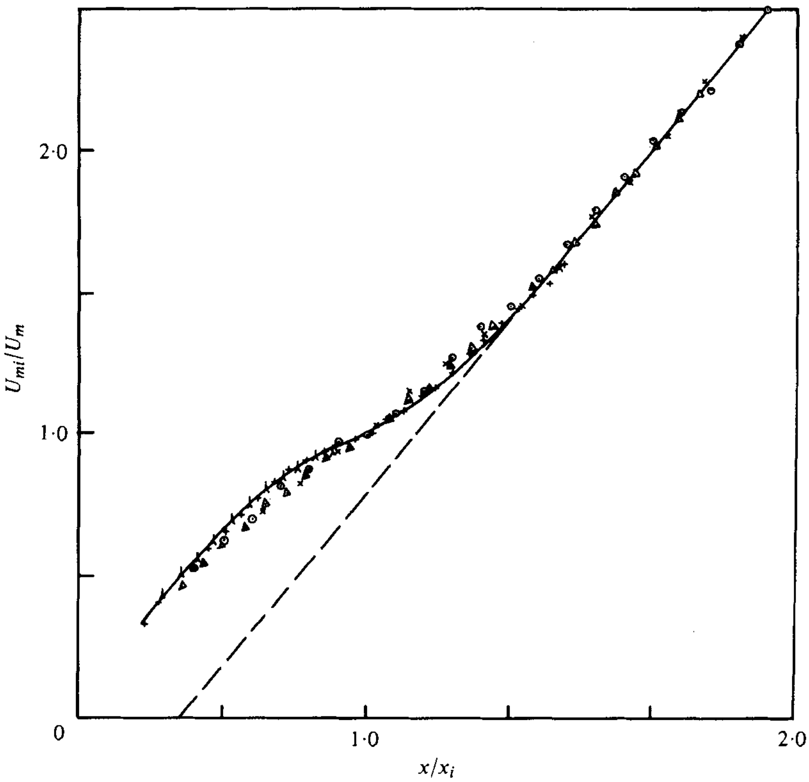


FIGURE 4. Collapse of maximum velocity results through circular jet to wall jet transition. For an explanation of the symbols see Figure 3.

in terms of the half velocity and maximum velocity positions and are compared with exponential and Görtler (1942) functions which approximate the free-jet velocity profile (Rajaratnam 1976); that is:

$$U/U_m = e^{-(0\cdot833\eta)^2}$$

and

$$U/U_m = (1+(\sqrt{2}-1)\eta^2)^{-2}$$

respectively, where

$$\eta = \frac{y-y_m}{y_{0\cdot5}-y_m}.$$

The distance y is measured perpendicular from the plane and suffixes 0·5 and m denote half and maximum velocity positions. On the inner side of the maximum velocity position near the solid surface a slight tendency for higher local velocities to occur in the early stages of interaction may be seen (for $h/D = 4\cdot0$, $x/D = 32$ and $h/D = 2\cdot0$, $x/D = 16$). In the later stages the velocities are below the free jet profiles. The former effect would be caused by the reduction of mixing rate of a nearly circular jet due to the plane before the jet has adopted the very much elongated form parallel to the plane. The latter effect would be due to the progressive thickening of the inner turbulent layer adjacent to the solid surface where the velocity profile is dominated by the shear on the surface rather than turbulent mixing with the undisturbed surrounding fluid.

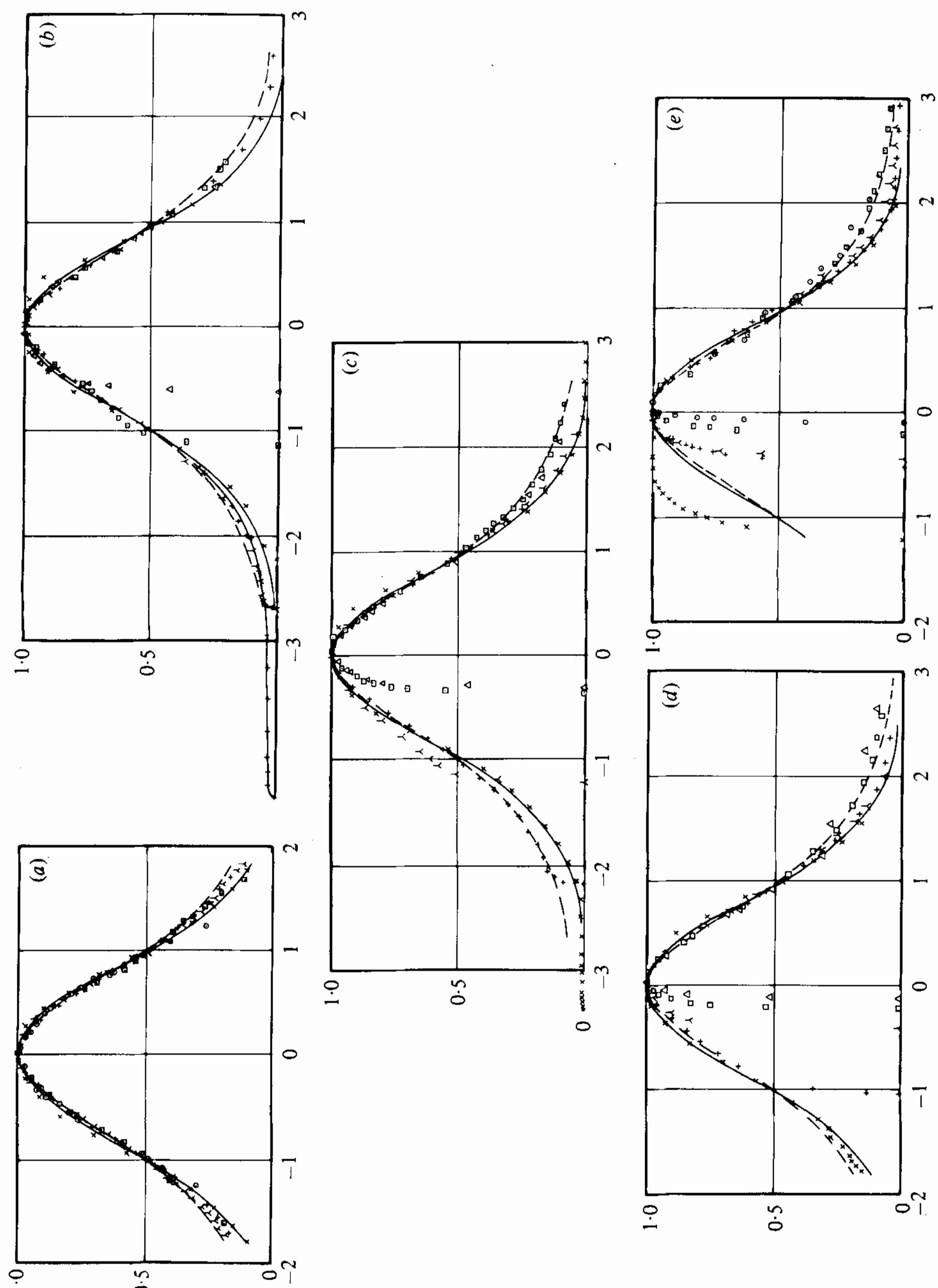


FIGURE 5. Velocity profiles perpendicular to the plane: ×, $x/D = 4{\cdot}5$; +, $x/D = 8{\cdot}0$; λ, $x/D = 16{\cdot}0$; ⊡, $x/D = 32{\cdot}0$; ⊙, $x/D = 48{\cdot}0$; △, $x/D = 64{\cdot}0$. (*a*) Unconfined free jet; (*b*) $h/D = 4{\cdot}0$; (*c*) $h/D = 2{\cdot}0$; (*d*) $h/D = 1{\cdot}0$; (*e*) $h/D = 0{\cdot}5$. ——, exponential profile; - - -, Görtler profile.

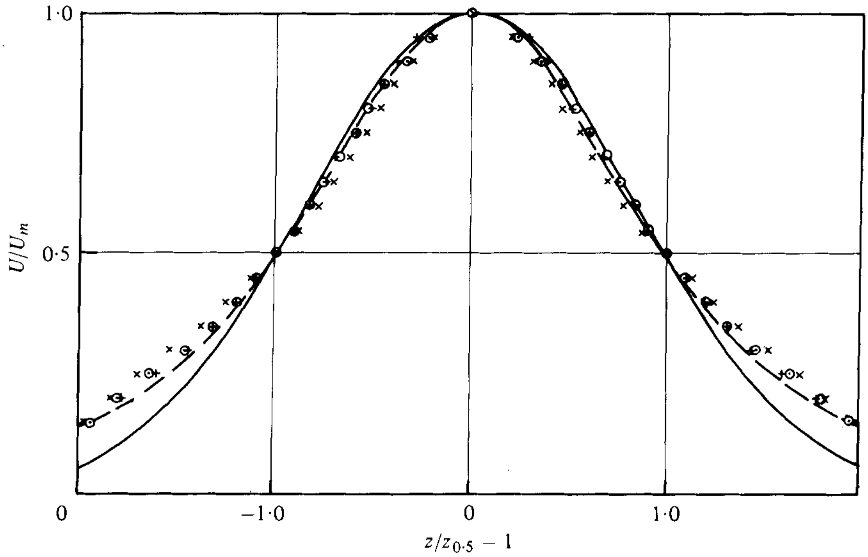
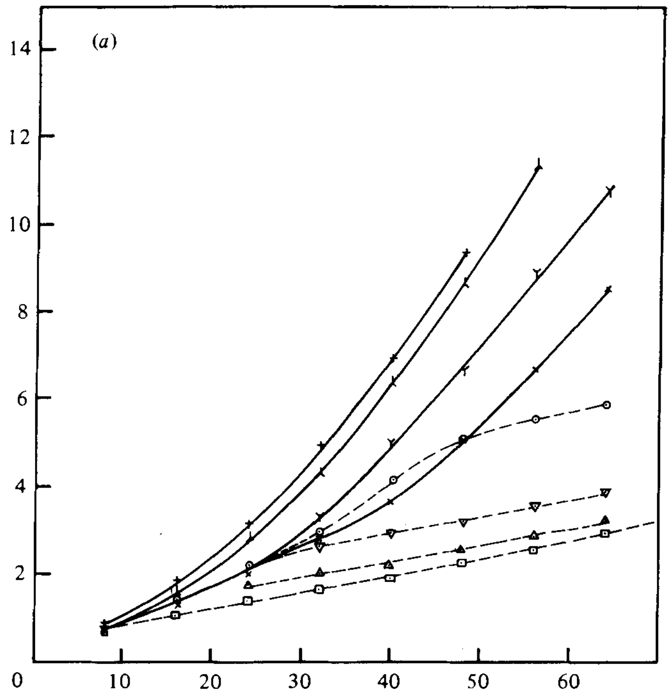


FIGURE 6. Velocity profiles parallel to plane ($U(z)$): +, $h/D = 4{\cdot}0$; ○, $h/D = 1{\cdot}0$; ×, $h/D = 0{\cdot}5$.

Distributions of local maximum mean velocity (u component for $\partial(U(y,z))/\partial y = 0$ at a given value of z) parallel to the plane are shown in figure 6, and are again generally similar to those of a free jet. It seems that there is a systematic tendency for the more-developed wall-jet profile (i.e. $h/D = 0{\cdot}5$ where the wall jet is substantially elongated as shown in figure 2 at $x/D = 48$) to give rather lower velocities in the inner region and higher velocities in the outer region. This slight departure from the free-jet form of mean velocity profile would be caused by the location of the maximum velocity position (in the z direction) close to the surface where wall shear effects would influence the mixing process. In the direction perpendicular to the wall the wall effects would not be so great and thus the velocity profiles when normalized resemble the free-jet form more closely.

The substantial increase in diffusion rate parallel to the plane is shown in figure 7 (a) which compares the transverse width and height scales (defined as the distance to the half maximum-velocity position, in the outer region for the height scale normal to the surface, L_y). As would be expected the influence of increasing clearance is to delay the departure of the scales from those for a free jet to a greater downstream distance. Comparison with the results of Newman *et al.* (1971) in the case for $h/D = 0{\cdot}5$ (figure 7b) shows that slightly larger values of L_z are measured here and slightly smaller values of L_y. However, Newman *et al.* used a jet emanating from a pipe with a small effective displacement from the wall due to the pipe thickness. The combination of this with the non-uniform turbulent velocity profile from a pipe thus seems to have influenced their measurements, the reduction of the effective nozzle size reducing the values of L_z and the small displacement increasing the value of L_y slightly compared with the present results. Rajaratnam & Pani (1974) obtained rather lower values of L_z than Newman *et al.* (1971), but for $x/D < 50$ where the present results were mainly obtained the trend of Rajaratnam & Pani's results suggests that a larger value of L_z would apply. Thus it does appear that the use of a fully turbulent pipe to provide the



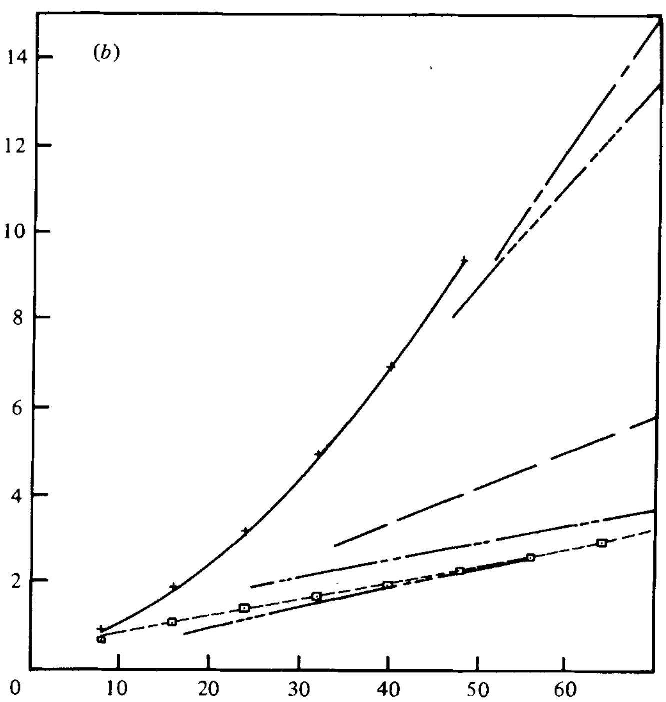


FIGURE 7. (*a*) Increase of jet scales with axial distance: ——, L_z/D; – – –, L_y/D. Present results: +, □, $h/D = 0{\cdot}5$; ⅄, △, $h/D = 1{\cdot}0$; Y, ▽, $h/D = 2{\cdot}0$; ×, ○, $h/D = 4{\cdot}0$. Of each pair of symbols, the first represents L_z/D, and the second L_y/D). (*b*) Increase of jet scales with axial distance ($h/D = 0{\cdot}5$): Present results: ——+——, L_z/D; ––––□––––, L_y/D. Newman *et al.* (1971); ——–——, L_z/D; ——– – ——, L_y/D. Rajaratnam & Pani (1974): ——– – – – ——, L_z/D; ——– – – ——, L_y/D. —— —— ——, free jet. Horizontal axis: x/D.

h/D	L_{y0}/D	x_{z0}/D	dL_z/dx	dL_y/dx	Comments
0·5	0·45	19·0	0·32	0·037	$x/D \geqslant 40$
1·0	0·85	21·0	0·33	0·036	$x/D \geqslant 40$
2·0	1·45	22·0	0·29	0·039	$x/D \geqslant 48$
4·0	3·2	25·5	0·23	0·046	$x/D \geqslant 48$
—	0	0	0·085	0·085	Wygnanski & Fiedler (1969), free jet
0·5	0·9	20·0	0·30	0·040	Newman *et al.* (1971)
0·5	0·1	12·0	0·23	0·046	Rajaratnam & Pani (1974)
0·5	0·35	—	—	0·065	Sigalla (1958), plane wall jet
0·5	0	—	—	0·080	Myers, Schauer & Eustis (1961), plane wall jet
0·5	0·68	—	—	0·068	Rajaratnam (1976), plane wall jet

TABLE 2. Apparent origin initial size and growth rates for turbulent jets.

jet by Newman *et al.* (1971) has led to lower results for L_z being obtained. It also appears that the present results show a rate of change of L_z with x close to that of Newman *et al.* (1971), whilst the rate observed by Rajaratnam & Pani (1974) was appreciably lower. It would not be expected that the flow structure at the nozzle exit would influence the spreading rate at such large distances downstream, although it would affect total flow momentum flux and absolute values of the length scales.

Rajaratnam & Pani (1974) show results for L_y at $h/D = 0{\cdot}5$ much more closely in agreement with those of the present work (see figure 7*b*). It may be seen that for the larger separations (figure 7*a*) ($h/D = 4{\cdot}0, 2{\cdot}0$) there is initially a region in which $L_y > L_z$, although further downstream this condition is reversed as the rate of diffusion parallel to the plane increases. This effect is due to the displacement of the nozzle above the plane and it may be seen from figure 8 that the point at which the maximum velocity position begins to move away from the surface at large x/D corresponds to the position where the rates of increase of scales adjust to those of the case with no clearance, (i.e. at $x/D \simeq 50$ for $h/D = 4{\cdot}0$ and at $x/D \simeq 30$ for $h/D = 2{\cdot}0$). Although it thus appears that at these positions the rates of increase of jet dimensions have nearly settled to the self-preserving values, the mean velocity distributions on the flow cross section would not resemble those of the self-preserving wall jet until a much later stage. In the case $h/D = 4{\cdot}0$ as figures 7(*a*) and 2 show, the jet still has nearly equal values of L_y and L_z, although the rates of change of these are shown by figure 7(*a*) to be very different, L_z increasing at approximately 7·5 times the rate of L_y.

The foregoing discussion shows that the influence of clearance space h/D is to shift the apparent origin of the jet, moving it further downstream of the nozzle in the case of the growth parallel to the plane and further upstream for growth normal to the plane. Table 2 shows this effect in detail in terms of the constants L_{y0} and x_{z0} in

$$(L_y/D) = (L_{y0}/D) + Ax/D$$

and

$$(L_z/D) = B(x/D - x_{z0}/D).$$

Although the apparent initial scale L_{y0} is somewhat smaller than the distance from the nozzle centre-line, it may be seen that the two values increase together and are of comparable magnitude. Thus, the apparent initial size of the jet at right angles to the plane may be associated simply with the displacement of the jet from the plane. The growth rates in this direction appear to increase slightly with clearance, although the values are generally somewhat less than those of Newman *et al.* and appreciably less than those of Rajaratnam & Pani for $h/D = 0{\cdot}5$. The growth rates are rather less than half those for a free jet and are also less than those for a plane wall jet. Even though the three-dimensional jet takes on such an elongated shape it appears from table 2 that the growth rates in L_y are appreciably modified by three-dimensional effects since they are appreciably smaller than those for plane wall jets. Thus the flow does not approximate the plane wall-jet behaviour even in the region near the maximum velocity position where L_y is measured. Table 2 also shows the very substantial increase of jet growth rate parallel to the plane, between 2·7 and 3·9 times the rate for a free jet and between 5·0 and 9·2 times the growth rate perpendicular to the plane. The rather low growth rate at $h/D = 4$ is probably caused by the three-dimensional transition for $x/D \leqslant 64$ and a larger rate would most likely be observed further from the nozzle. The rate of increase of apparent origin displacement x_{z0}/D with clearance h/D is only of order 2 and appears slower than the increase of distance at which the plane would intersect the edge of a free jet ($dx/dh = 6{\cdot}2$, if the free jet is considered to have an 'edge' at $U/U_m = 0{\cdot}1$). This is probably because the interaction transition process at small clearances requires a larger distance because of the combination of transition from initial smooth core to fully turbulent flow and the circular to wall jet changes which must both take place. This results in a very large displacement of the apparent origin of the wall jet ($x_{z0}/D = 21{\cdot}0$ at $h/D = 1{\cdot}0$) which is much greater than the distance at which the plane would just begin to intersect an undisturbed free jet ($x/D = 3{\cdot}1$ at $h/D = 1{\cdot}0$, taking the jet edge at $U/U_m = 0{\cdot}1$). At larger clearances the increase of outward displacement of the apparent origin with clearance is slower than might be expected because the cases where the interaction first occurs with a fully turbulent jet then require a relatively smaller transitional distance. At $h/D = 4$ the apparent origin for the sideways spreading corresponds to the position where the plane would intersect a point $U/U_m = 0{\cdot}21$ of an undisturbed free jet, which further emphasizes this aspect. We may conclude that where the first interaction takes place in the initial region of the original circular jet a relatively longer transition to three-dimensional wall jet occurs than where first interaction is in the fully turbulent or self-preserving region of the circular jet.

3. Turbulent properties of the wall jet

A series of measurements of the turbulent characteristics of the jet was made to determine the mechanism by which the greatly enhanced diffusion rate parallel to the plane was produced. The measurements were made using a Thermo-Systems true r.m.s. meter and a Hewlett Packard digital correlator/spectrum analyser. As the lower limiting frequency of the voltmeter was 0·1 Hz and the upper-frequency response limitation of the hot-wire equipment was 20 KHz or better the measurements were not significantly influenced by the performance of the measuring system. Measurements were taken in the range $x/D < 32$ to avoid entering a low-mean-velocity region in the flow where the lower frequency limit can be significant as discussed by Wygnanski & Fiedler (1969). At the outermost measuring position ($y/D \simeq 5$) at the most downstream position ($x/D = 32$) where the low-frequency limitation would be more severe,

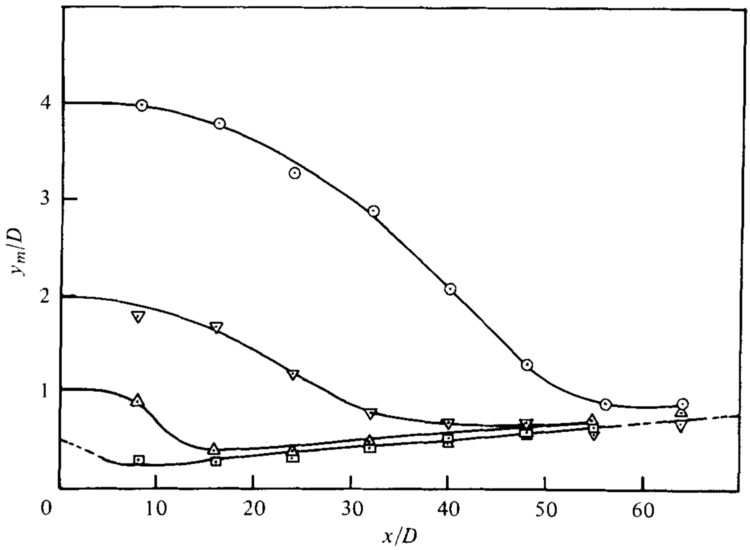


FIGURE 8. Location of maximum velocity position above surface: □, $h/D = 0{\cdot}5$; △, $h/D = 1{\cdot}0$; ▽, $h/D = 2{\cdot}0$; ○, $h/D = 4{\cdot}0$.

it was estimated that the error due to the low-frequency cut off was approximately 1 %. This is much less than other errors associated with the use of inclined wire probes (see § 2).

The distribution of axial component turbulence level is shown in figure 9. The results for the unconfined free jet show the pattern of growth in turbulence level with downstream distance and adjustment of profile shape which has often been observed previously for the transition region, (e.g. Bradshaw *et al.* 1964). The most downstream position ($x/D = 32$) approximately corresponds to the establishment of self-preservation in terms of the turbulence level, which reaches a maximum of approximately $0{\cdot}25\,U_m$. A slightly higher value ($0{\cdot}28\,U_m$) is observed at large distances downstream, (e.g. see Wygnanski & Fiedler 1969). At the largest nozzle/plane clearance ($h/D = 4{\cdot}0$), a progressive reduction of streamwise turbulence level occurs over the inner half of the flow, by up to 25 % at $x/D = 32$ and by 10 % approximately at $x/D = 16$. At closer clearances ($h/D = 2{\cdot}0, 1{\cdot}0$) similar effects may be observed, and larger reductions of turbulence level then occur closer to the nozzle, (e.g. by 25 % at $x/D = 8{\cdot}0$ for $h/D = 1{\cdot}0$). Also, it may be observed that far downstream the reduction of turbulence level became smaller again outside the immediate wall layer where it drops sharply to zero at the surface. For example reductions of only 12 % occur at $h/D = 1{\cdot}0, x/D = 32$. This effect is also observed when the nozzle is adjacent to the plane ($h/D = 0{\cdot}5$), the turbulence level at $x/D = 32$ being only slightly reduced whilst larger reductions occur closer to the nozzle. The inner turbulent profile at $h/D = 0{\cdot}5$ in the potential core region ($x/D = 3{\cdot}5$) shows a reduction on the potential core side of the wall shear layer, but has a sharp maximum adjacent to the wall, more resembling the distribution in a turbulent boundary layer, with a maximum level value of about 14 %. These observations show that in the transitional region where the circular jet adjusts to a three-dimensional wall jet lower turbulence levels occur. These reductions are strongest towards the inner edge of the flow (i.e. towards the plane) and reach a maximum

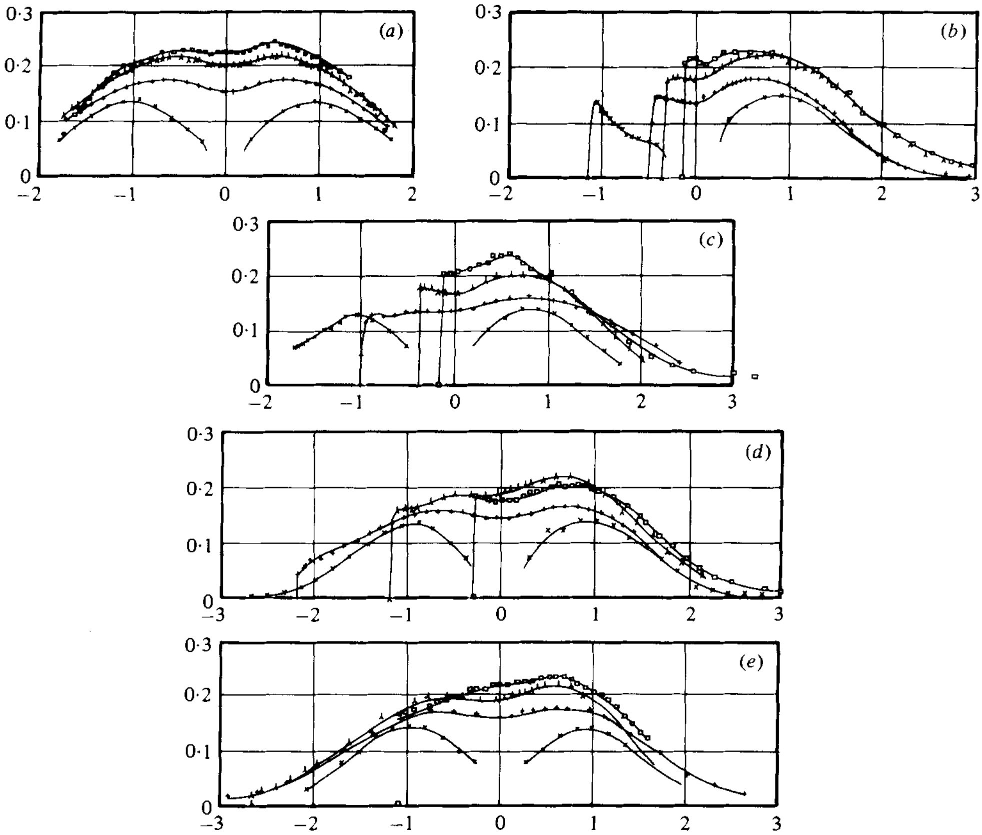


FIGURE 9. Streamwise turbulence level distributions $((\overline{u'^2})^{\frac{1}{2}}/U_m)$ $\times$, $x/D = 4{\cdot}5$; $+$, $x/D = 8{\cdot}0$; λ, $x/D = 16{\cdot}0$; $\square$, $x/D = 32{\cdot}0$. Horizontal scales: $(y-y_m)/(y_{0\cdot5}-y_m)$. Vertical scales: $(\overline{u'^2})^{\frac{1}{2}}/U_m$. (a) Unconfined jet; (b) $h/D = 0{\cdot}5$; (c) $h/D = 1{\cdot}0$; (d) $h/D = 2{\cdot}0$; (e) $h/D = 4{\cdot}0$.

reduction of about 25 % compared with the unconfined jet at the same position. In general the levels in the outer region of the flow are not significantly affected. However, further downstream where the three-dimensional wall jet is established the turbulence levels return to values similar to the unconfined free jet. Newman *et al.* (1971) also observed that far downstream the turbulence level reached approximately 24 % as in the case of an unconfined jet. It may be concluded therefore that reductions of turbulence level occur only in the transitional process to developed wall-jet flow. Such a reduction of level might thus account for the displacement away from the nozzle of the apparent origin of the wall jet due to the reduced rate of mixing (presuming that other turbulent mixing parameters undergo similar reductions), but would not explain the growth rates observed for the developed wall jet. Indeed, since the jet diffusion normal to the plane is approximately halved whilst that parallel to the plane is increased by a factor of three it is clear that the wall jet behaviour must be caused by a significant difference in the transverse mixing effects.

The transverse turbulence levels for the wall jet case are compared with that in a free unconfined jet in figure 10. The maximum value parallel to the plane is increased by 15 %, whilst that perpendicular to the plane is increased by 12 %. Therefore it is

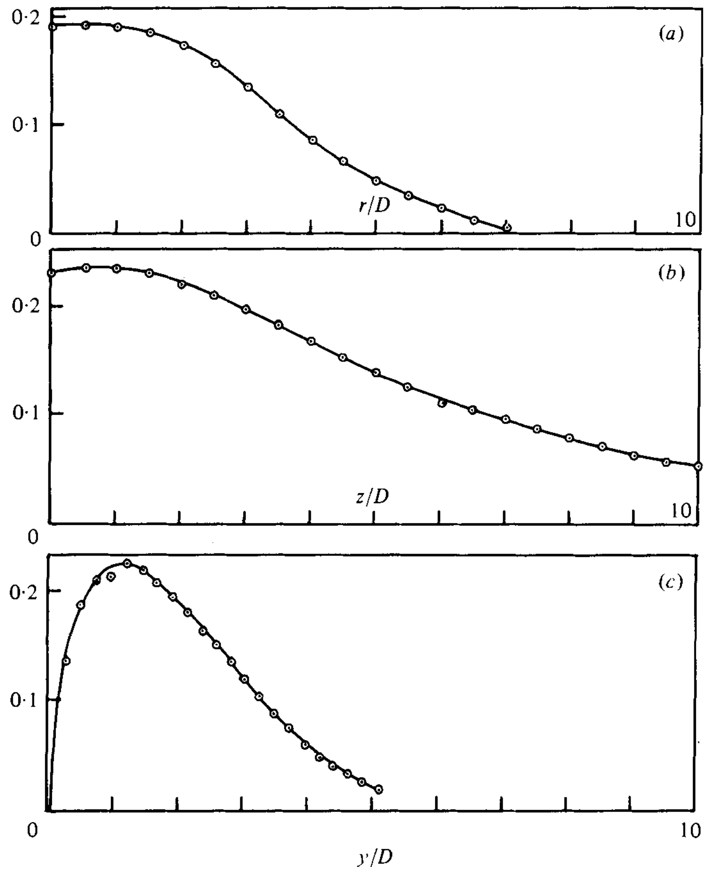


FIGURE 10. Distribution of transverse turbulence level. (a) Unconfined jet, $(\overline{v'^2})^{\frac{1}{2}}/U_m$ *vs.* r/D; (b) jet over plane, $(\overline{w'^2})^{\frac{1}{2}}/U_m$ *vs.* z/D; (c) jet over plane, $(\overline{v'^2})^{\frac{1}{2}}/U_m$ *vs.* y/D; $x/D = 32{\cdot}0, h/D = 0{\cdot}5$.

concluded that the influence of the plane in producing the different diffusion rates cannot be explained simply on the basis of the maximum level of the transverse fluctuations as both components are increased. The influence of the plane must therefore be to modify the structure of the turbulence in some way.

Reynolds stress distributions were measured using the cross wire probes and are shown in figure 11. For the unconfined jet case there is very good agreement with the results of Wygnanski & Fiedler (1969) which were shown to be consistent with the mean velocity profile although this is not a highly sensitive indicator of precision. Of the turbulent structural parameters derived from the Reynolds stress, the mixing length was found to reflect changes in structure most clearly. The ratio $\overline{u'v'}/\overline{q^2}$ took on a value of 0·10, very close to that of a free jet, at the one position in the wall jet where all turbulent velocity components were measured. However more extensive measurements of this parameter would be needed to confirm that the stress structure of the wall and unconfined jets are similar in this respect. In the wall jet results the maximum stress component on planes normal to the solid surface (i.e. $-\rho\overline{u'w'}$) is increased by 50 % compared to the free jet, whilst the component on planes parallel to the solid surface (i.e. $-\rho\overline{u'v'}$) has a maximum normalized stress almost equal to

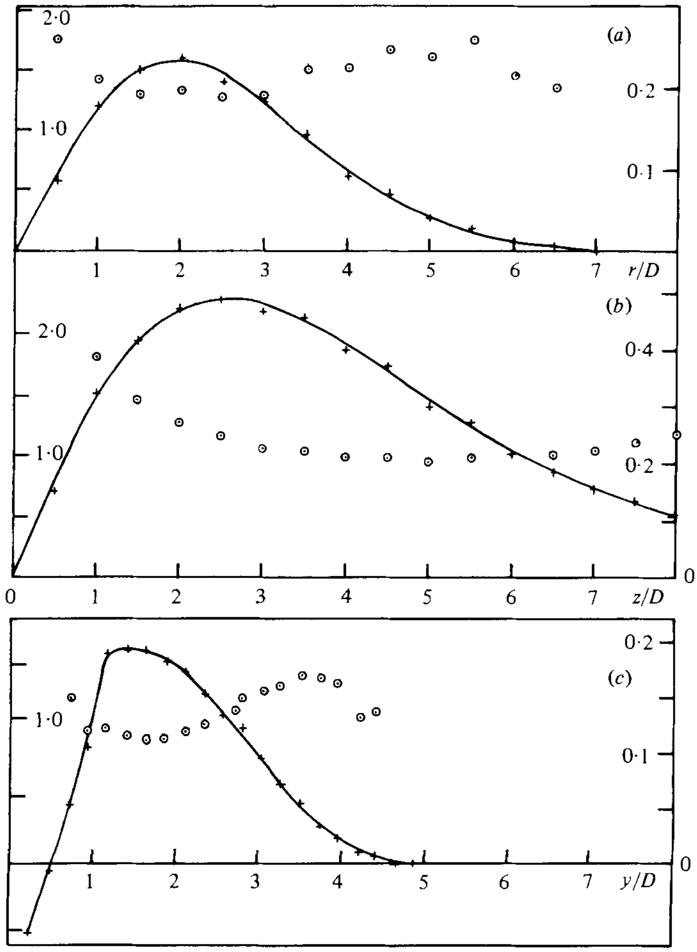


FIGURE 11. Distribution of Reynolds stress and mixing length. (a) Unconfined jet, $\overline{u'v'}/U_m^2$ *vs.* r/D and l_r/L_r *vs.* r/D. (b) Jet over plane, $\overline{u'w'}/U_m^2$ *vs.* z/D and l_z/L_z *vs.* z/D. (c) Jet over plane, $\overline{u'v'}/U_m^2$ *vs.* y/D and l_y/L_y *vs.* y/D. +, Reynolds stress × 10^2 (left-hand scale); ○, mixing length (right-hand scale); $x/D = 32{\cdot}0$, $h/D = 0{\cdot}5$.

that of the free jet. In the latter case (figure 11*c*) it is seen that the turbulent stress reverses in sign at the maximum velocity position ($y/D = 0{\cdot}5$). Figure 11 also shows the mixing length distributions, as implied on the basis of the Reynolds stress observations,

$$l_y = \left(\frac{-\overline{u'v'}}{(\partial U/\partial y)^2}\right)^{\frac{1}{2}}, \quad l_z = \left(\frac{-\overline{u'w'}}{(\partial U/\partial z)^2}\right)^{\frac{1}{2}}.$$

It seems that there is some variation across the jet, and a much larger mixing length parallel to the solid ground plane is implied compared to that at right angles to the plane. It should be noted that the mixing length as usually defined in this way does not represent a directly measured physical scale of the motion, but is in effect a representation of the flow stress. The assumption of a constant mixing length is the basis of the exponential mean-velocity profiles (Tollmien 1926) which have already been shown to be in good agreement with the observed mean-velocity profiles. For the circular jet case the Tollmien analysis relates the transverse scale of the jet to the mixing length by:

$$\frac{L_r}{x} \sim (l_r/L_r)^2 = \frac{-\overline{u'v'}}{(\partial U/\partial r)^2 L_r^2}$$

where l_r is the radial mixing length and L_r the radial distance to the half maximum mean-velocity position. That is, the angular spread rate of the jet is proportional to the square of the ratio of mixing length to transverse scale (see, for example, Tollmien 1926; Rajaratnam 1976). Rajaratnam (1976) gives a value of $l_r/x = 0{\cdot}017$ (based on the mean velocity data of Trupel (1915), which is close to the present observations), whilst the value of (L_r/x) is 0·084 (Corrsin 1946) or 0·085 (Wygnanski & Fiedler 1969). Hence for a circular free jet the ratio $l_r/L_r = 0{\cdot}20$. This compares well with the present free jet data (figure 11*a*) which gives a mean value of $l_r/L_r = 0{\cdot}22$ across the jet.

In the case of the wall jet, which becomes very elongated in cross-section, the relation between Reynolds stress and spreading rate would be somewhat modified due to the three-dimensional nature of the flow. The observed Reynolds stresses (figure 11) for the wall jet at $x/D = 32$ lead to a ratio $(l_z/L_z)^2/(l_y/L_y)^2 = 4{\cdot}0$, whereas the observed ratio of spreading rates for the wall jet is at this position (figure 7*a*):

$$[(dL_y/dx)/(dL_y/dx)] = 5{\cdot}9.$$

It appears therefore that the measurements of Reynolds stress are in a rather lower ratio than the ratio of transverse to normal spread rate for the wall jet. The wall jet at the position where observations were made ($x/D = 32$, $h/D = 0{\cdot}5$) is not in a fully developed state and the mean velocity distributions are still in a state of transition to the wall jet form although they are nearer to the wall jet form than the circular jet. Further, an identical form of relation between Reynolds stress and spreading rate for the circular jet and for the height and width spreading of the wall jet is not expected because of the three-dimensional form of the latter. However, it may be said that the ratio of observed Reynolds stresses is approximately in proportion to the observed spreading rates at $x/D = 32$. The higher observed ratio for the spreading rates is most probably due to three-dimensional effects on the width spreading across the plane, where the contours of equal mean velocity on the flow cross-section are most sharply curved.

A number of observations of spectra of the velocity component fluctuations were made, but in the main these showed very little difference in the form from those of the free jet provided the observations were compared on the basis that the local mean velocities were modified in the interacting jet condition. Close to the jet in the transitional region the spectra in the surface interacting flow showed a slight tendency for the higher frequency components to be strengthened and the lower frequency components to be weakened compared to the free-jet spectrum (see figure 12). Such an effect would result simply from the physical restriction imposed by the plane preventing the formation of the larger turbulent structures which occur in the free jet. However, the difference of spectral form is quite small even at this position in the transitional region ($x/D = 8$), whilst further downstream ($x/D = 32$) the spectra in the presence of the plane appeared virtually unaltered. Transverse component spectra also showed negligible influence due to the plane. The probes used in these experiments were not

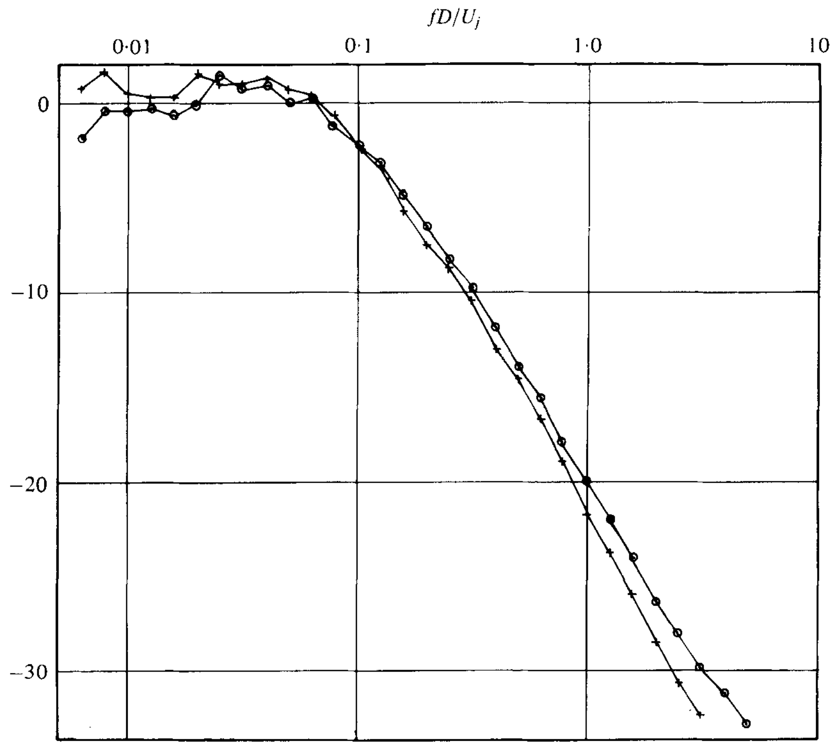


FIGURE 12. Spectra of streamwise velocity fluctuations: +, unconfined free jet; ⊙, jet over plane; $h/D = 0{\cdot}5$, $x/D = 8{\cdot}0$, measured at position where $U/U_m = 0{\cdot}5$ (outer edge of flow in case of jet over plane). Horizontal scale: fD/U_j. Vertical scale: $dBreU^2D/U_j$.

designed to be traversed accurately within the immediate boundary layer on the wall as Rajaratnam (1976) indicated that the wall stress played a very small role in the decay of the jet. The spectra in these boundary layer regions were not therefore observed. Measurements of spectra were made to within a distance of $0{\cdot}1\,D$ from the wall, but did not show any significant variation in spectral form from those in the body of the flow.

To demonstrate the manner in which the plane and flow interact correlation measurements were made between the transverse components of velocity taking a fixed reference position for the component perpendicular to the plane (v_1) at the centre of the flow and at the position where maximum Reynolds stress was observed. A moving X-wire probe was arranged to sense the transverse velocity (w) component parallel to the plane, and a series of traverses away from the plane in the y direction were made at different distances (z) from the centre of the flow. The results are shown in figure 13. It is seen that there is a good correlation between the outflow (or inflow) velocity along the plane and the inflow (or outflow) velocity at the reference point in the centre of the flow, whilst in the outer region the component parallel to the plane is directed towards (or away from) the plane of symmetry. These results show that eddying motions in the plane perpendicular to the jet interact with the surface such that disturbances perpendicular to the solid plane are deflected along the plane in the manner of flow at a stagnation point in the cross-flow plane. The scale of this

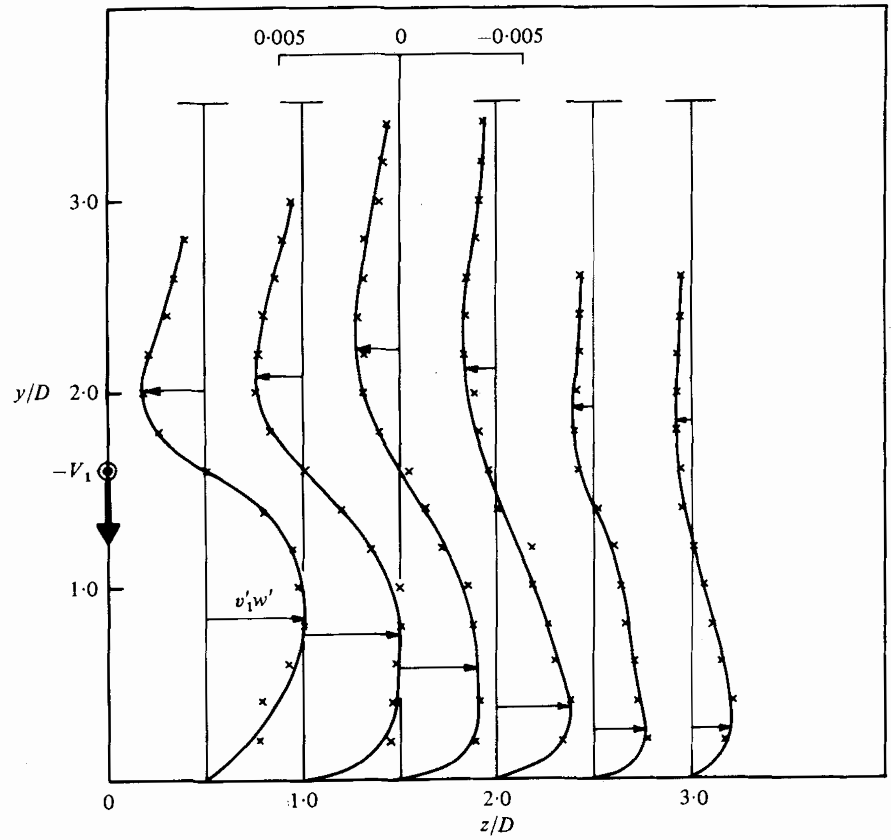


FIGURE 13. Cross-correction of transverse velocity components

$$\left[\frac{V_1' \,.\, w'(y,z)}{U_m^2}\right]; \quad x/D = 32{\cdot}0,\ h/D = 0{\cdot}5.$$

interacting-flow pattern is quite large in the z direction along the plane, the peak correlation only halving in magnitude at $z = 2{\cdot}5\,D$, whilst in the direction perpendicular to the plane the correlation becomes zero at about $y/D = 1{\cdot}4$. Thus we see that the surface interacts with normal component inward disturbances so that symmetrical outward motions are produced along the plane and vice versa, the scale of the motion being larger along the plane than perpendicular to it. In the outer region of the flow away from the plane the entrainment motion is somewhat weaker, the peak correlation being approximately 60 % of the maximum in the region of outflow along the surface.

4. Conclusion

The displacement of the circular nozzle from the solid ground plane over which the jet is diffusing has the effect of increasing the distance of the apparent origin of the developed wall jet from the nozzle. In terms of the transverse width scale of the jet this distance increases rather slowly with the displacement due to a longer transitional process at close nozzle/plane separations which causes a fairly large outward movement of the apparent origin at close separations. In terms of the maximum velocity the apparent origin is much nearer to the nozzle at close separations and moves away from the nozzle more uniformly with increasing separation. At right angles to the plane the development of the jet over the wall appears to have an initial transverse height scale somewhat smaller than the distance between the jet axis and the plane. The developed wall jet has a rate of decay of peak velocity comparable with that of a free jet, but has a rate of increase of width scale parallel to the plane which is approximately 3·7 times that of the free jet. The increase of height scale perpendicular to the plane is less than half that of the free jet.

Turbulence levels in the transition region between circular and wall jets are reduced by up to 25 %, but subsequently return in the wall jet to values close to those in a free jet. Reynolds stresses lead to the implied mixing length parallel to the plane being 5·7 times that perpendicular to the plane at $h/D = 0{\cdot}5$, $x/D = 32$. This difference in Reynolds stress is approximately consistent with the spread rates of the jet at this position, being 5·9 times greater parallel to the plane than normal to the plane. However, the spread rates are not in exact proportion to the Reynolds stresses and it appears that significant three-dimensional effects exist in the wall jet due to its greatly elongated cross-sectional form. In the fully developed wall jet these ratios of spreading rate would increase, the latter to approximately 8·5 : 1. Correlation measurements show the existence of a large-scale motion in the plane at right angles to the flow in which the normal component motions towards the plane produce strong outflow motions along the plane in a symmetrical manner about the plane of symmetry. This motion has a scale parallel to the plane which is greater than that normal to the plane and provides a physical mechanism for the much greater mixing rate parallel to the plane.

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